ESG Attribution: A Risk-Parity Approach to Effective Risk Control

This paper introduces a framework to assess the impact of active ESG investment strategies, which internally integrate ESG analysis or asset owner constraints, unlike passive approaches relying on external ESG indices. Both methods reduce the investable universe, but active strategies require tailored risk-return evaluation. Unlike traditional proportional reallocation –which overlooks additional risk– the risk-parity approach redistributes excluded assets to ensure balanced risk contributions across the remaining portfolio. This approach offers three key benefits: equal incremental risk contribution, preservation of the benchmark's risk structure, and isolation of ESG effects, ensuring performance differences reflect only ESG strategy, not unintended risk shifts. The framework empowers asset managers to align financial and sustainability goals, ensuring ESG integration enhances –rather than disrupts– index risk-return trade-off.

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INTRODUCTION

This paper introduces a framework to assess the impact of active investment strategies, which internally integrate custom constraints or analytical processes, such as ESG analysis or asset owner limitations. Unlike passive approaches relying on external indices, active strategies require tailored risk-return evaluation. The risk-parity approach redistributes excluded assets to ensure balanced risk contributions across the remaining portfolio. This approach offers three key benefits: equal incremental risk contribution, preservation of the benchmark's risk structure, and isolation of the strategy's effects, ensuring performance differences reflect only the active constraints, not unintended risk shifts.

While the framework is illustrated using ESG strategies, its principles and techniques are universally applicable to any active management mandate that excludes specific sectors, securities, or asset classes from a benchmark. Active strategies are defined as those that incorporate custom constraints or analytical processes—whether internally developed or imposed by the asset owner—rather than relying on externally defined benchmarks. This contrasts with passive strategies, where constraints are determined by predefined indices (*e.g.*, ESG indices, sector-specific indices, or style indices). In both cases, the contraction of the investable universe alters the benchmark's risk-return profile, necessitating a rigorous approach to reallocation and performance attribution.

The proliferation of ESG indices provides asset managers with readily available benchmarks that reflect broad sustainability criteria. However, many asset owners and managers adopt custom ESG strategies that go beyond standard indices, either by excluding additional sectors (*e.g.*, fossil fuels, controversial weapons) or by applying proprietary ESG scoring methodologies. Similarly, non-ESG active strategies—such as those excluding sin stocks, leveraged securities, or specific geographies—face analogous challenges in benchmark construction and performance attribution. While standard indices offer a convenient proxy for market segments, custom indices based on the actual investable universe are often necessary to accurately reflect a manager's constraints. This paper's risk-parity reallocation framework addresses this need by ensuring that excluded assets are redistributed in a risk-controlled manner, regardless of whether the constraints are ESGrelated or stem from other active management decisions. This approach is particularly valuable when no off-the-shelf index adequately represents the manager's universe, or when the cost and complexity of maintaining a custom index are justified by the strategy's specificity.

Implementing a risk-parity reallocation framework delivers substantial benefits for risk management and transparency. However, the approach demands robust data on asset volatilities and correlations, as well as the flexibility to dynamically adjust allocations in response to changing constraints. For asset owners, this may entail additional resources for data management, reporting, and stakeholder communication, particularly when returns diverge from widely followed market indices. Moreover, the approach is most justified for strategies where standard indices are insufficient—either because the investable universe is highly customized (*e.g.*, exclusion of multiple sectors) or because the manager's constraints are not reflected in available benchmarks. In cases where existing closely align with the strategy, the incremental value of a custom risk-parity approach must be weighed against its costs. However, for strategies with unique or evolving constraints, this framework provides a rigorous alternative to proportional reallocation, ensuring that performance attribution remains uncontaminated by unintended risk biases.

The remainder of the paper is organized as follows. Section 1 introduces the risk parity–based allocation framework. Section 2 presents the methodology used to quantify the impact of an active ESG strategy on the benchmark within a risk parity setting. Section 3 provides an empirical illustration of ESG Attribution and discusses the strengths and limitations of the proposed approach.

SECTION 1: RISK PARITY ALLOCATION

Modern portfolio theory has long emphasized the trade-off between risk and return as the central problem of asset allocation. Since the seminal work of Markowitz (1952), portfolio construction has predominantly relied on capital-based allocation frameworks, most notably mean–variance optimization and fixed-weight strategies such as balanced equity–bond portfolios. While these approaches aim to achieve diversification through capital allocation, it has been widely recognized that they often fail to diversify risk effectively.

In response to these shortcomings, risk-based allocation methodologies have gained prominence in both academic research and institutional investment practice. Among them, the risk parity approach has emerged as a compelling alternative to traditional capital-weighted portfolios. The core principle of risk parity is to allocate assets such that each component contributes equally to the total portfolio risk, typically measured by volatility (Qian, 2005; Maillard, Roncalli, & Teïletche, 2010). By shifting the focus from capital allocation to risk contribution, risk parity offers a structurally different notion of diversification—one that is explicitly aligned with the portfolio's underlying risk drivers.

The theoretical foundation of risk parity lies in the decomposition of portfolio risk into marginal and total risk contributions. Rather than relying on expected returns, which are notoriously difficult to estimate with precision, risk parity portfolios are constructed using estimates of asset volatilities and correlations (Choueifaty & Coignard, 2008). Assets with lower volatility are therefore assigned larger capital weights, while more volatile assets receive smaller allocations. This mechanism often leads to portfolios that diverge substantially from traditional benchmarks and may involve the use of leverage to achieve return objectives comparable to equity-heavy portfolios.

When we assume that all assets or segments in the benchmark are perfectly correlated (i.e., pairwise correlations equal to one), the portfolio weights that ensure risk parity are given by

$$w_i^{\rho=1} = \frac{1/\sigma_i}{\sum_{i=1}^{N} 1/\sigma_i} \tag{1}$$

Under this assumption, it can be shown that the risk contributions of all assets or segments within the portfolio are identical, *i.e.,*

$Contrib_i = Contrib_j$ which implies $w_i\rho_{i,B}\sigma_i = w_j\rho_{j,B}\sigma_j \ \ \forall i,j$

Given that $\sum_{i=1}^{N} w_i = 1$ and $\rho_{i,j} = 1$, it directly follows that

$$w_i = \frac{1/\sigma_i}{\sum_{i=1}^{N} 1/\sigma_i}$$

When correlations differ from unity, the risk-parity weights are instead defined as

$$w_i^{\rho=a} = \frac{1/\rho_{i,B}\sigma_i}{\sum_{i=1}^{N} 1/\rho_{i,B}\sigma_i} \quad (2)$$

ILLUSTRATION

Table 1 below illustrates that the contribution to total volatility of each sector is identical when portfolio weights are derived from Equations 1 and 2. Given that the correlations are not equal to unity, the percentages derived from Equation 1 do not yield an equal risk contribution across all sectors. In contrast, the allocation provided by Equation 2 does ensure an equal risk contribution for each sector equal to 0.71.

We observe that assets with a low marginal contribution to portfolio risk receive significantly higher weights under a risk-parity approach compared to their initial benchmark allocations. For example, in the Utilities sector—where the marginal risk contribution is 5.44%—the benchmark weight is 5.41%. Under risk parity, this weight increases to 8.79% (assuming a correlation of 1) and 13.09% (assuming a correlation of 0.42), demonstrating a substantial reallocation.

Conversely, sectors with a high marginal risk contribution see their weights reduced relative to the initial benchmark. This is evident in the Industrials sector, where the marginal risk contribution is 9.29%. As a result, its portfolio weight decreases from 15.30% to 11.26% (correlation of 1) and 7.67% (correlation of 0.93), reflecting a systematic underweighting.

Additionally, the risk-parity benchmark exhibits lower overall volatility (7.13%) than the initial benchmark (8.24%), due to its more balanced risk distribution across sectors.

In the following section, we use the risk-parity framework to construct a new index that incorporates an ESG strategy. The proposed methodology ensures equal risk incremental contributions when reallocating assets or segments excluded through the ESG screening process.

SECTION 2: ESG ATTRIBUTION UNDER RISK-PARITY

Environmental, Social, and Governance (ESG) strategies may differ materially from those employed in the construction of ESG indices when the strategy is defined by the asset owner or asset manager, rather than by an index provider. In such instances, the approach is best characterized as an active ESG strategy, in contrast to a passive ESG strategy, wherein the investment universe and constraints are dictated by a predefined ESG index.

Any ESG strategy entails a reduction in the investable universe. A critical consideration thus arises regarding the reallocation of index weights associated with excluded securities or segments toward the remaining ad-

Table 1: Risk-Parity Weights for Correlations Equal to 1 or a

	w_i	$\rho_{i,B}$	σ_i	$\rho_{i,B}\sigma_i$	$w_i\rho_{i,B}\sigma_i$	$w_i^{\rho=1}$	$w_i^{\rho=a}$	$w_i^{\rho=1}\sigma_i$	$w_i^{\rho=a}\rho_{i,M}\sigma_i$
Consumer Discretionary	13.86	0.85	12.94	10.98	1.52	8.72	6.49	0.96	0.71
Consumer Staples	10.51	0.59	10.74	6.35	0.67	10.50	11.22	0.67	0.71
Energy	5.02	0.59	12.36	7.24	0.36	9.13	9.84	0.66	0.71
Financials	21.77	0.69	11.89	8.16	1.78	9.49	8.74	0.77	0.71
Health Care	7.41	0.74	13.36	9.89	0.73	8.44	7.20	0.84	0.71
Industrials	15.30	0.93	10.02	9.29	1.42	11.26	7.67	1.05	0.71
Information Technology	7.07	0.62	9.23	5.71	0.40	12.23	12.48	0.70	0.71
Materials	8.29	0.90	10.65	9.57	0.79	10.60	7.44	1.01	0.71
Telecom Services	5.35	0.43	10.40	4.50	0.24	10.85	15.83	0.49	0.71
Utilities	5.41	0.42	12.83	5.44	0.29	8.79	13.09	0.48	0.71
Total	**100.0**	**1.00**	**8.24**		**8.22**	**100**	**100**	**7.62**	**7.13**

missible assets. Grégoire (2019) proposes a methodology whereby the weights of excluded securities or segments are redistributed proportionally across the remaining portfolio holdings.

The methodology presented in this paper diverges from proportional reallocation and instead emphasizes risk-based portfolio control within the reallocation process. Specifically, the objective is to reallocate the weights of excluded positions across admissible securities such that the reallocated weight contributes equally to the incremental risk of each asset or segment within the index. Unlike traditional approaches that reallocate excluded weights in proportion to initial holdings, the proposed model ensures an equal-risk contribution allocation among admissible assets or segments, thereby fostering a more balanced risk distribution across the index.

The impact of an ESG investment strategy is quantified as the differential in returns between (i) the benchmark adjusted for ESG-driven reallocation and (ii) the original benchmark.

To construct the ESG-adjusted benchmark, the weights of excluded securities or segments must be redistributed among the remaining eligible assets. This redistribution is executed within a risk parity allocation framework, ensuring that the reallocated capital contributes uniformly to index risk.

Let:

> w_i denotes the initial weight of asset *i* in the original benchmark or index,
>
> w_i^{NESG} denotes the weight of excluded (non-ESG) assets, $w_i^{NESG} = 0$ when the asset is not excluded
>
> w_i^{ESG} denotes the weight of asset *i* following the reallocation of non-ESG-compliant assets

For the original benchmark, the weights satisfy:

$$\sum_{i=1}^{N} w_i = 1$$

The objective of the reallocation of w_i^{NESG} is twofold:

- The incremental risk contribution of the reallocated weight w_i^{NESG} must be identical for all eligible assets, and,
- The total portfolio weight after reallocation must sum to unity:

$$\sum_{i=1}^{N} w_i^{ESG} = 1$$

<u>Risk-Parity-Based Reallocation Rule</u>

When constructing an ESG-adjusted index, the exclusion of non-ESG-compliant assets necessitates a reallocation of their weights among the remaining eligible securities. A proportional reallocation may inadvertently alter the risk profile of the index, potentially concentrating risk in a subset of assets. To address this challenge, a risk-parity-based reallocation rule is proposed. This methodology ensures that the incremental risk contribution of the reallocated weights is uniform across all eligible assets, thereby preserving the index risk balance.[1]

Let p_i denote the proportion of the excluded weight w^{NESG} allocated to asset *i*. The new ESG-adjusted index weight for asset *i*, denoted w_i^{ESG}, is defined as:

$$w_i^{ESG} = w_i + p_i \cdot w^{NESG} \tag{3}$$

where:

> w_i is the weight of asset *i* in the benchmark
>
> w_i^{ESG} is the weight of asset *i* after reallocating non-ESG-compliant assets,
>
> w^{NESG} is the weight of excluded (non-ESG) assets.

The allocation proportions p_i are determined by imposing the condition of equal incremental contributions to benchmark risk. This condition is expressed as:

$$p_i \cdot w^{NESG} \cdot \rho_{i,\mathrm{B}} \cdot \sigma_i = \text{constant} \qquad \forall i,$$

where:

> σ_i is the volatility of asset *i*,
>
> $\rho_{i,\mathrm{B}}$ is the correlation between asset *i* and benchmark.

The allocation proportions p_i must satisfy the normalization constraint:

$$\sum_{i=1}^{N} p_i = 1$$

Under the assumption that asset-to-benchmark correlations $\rho_{i,\mathrm{B}}$ remain unchanged between the original and

ESG-adjusted index,[2] the risk-parity allocation coefficients p_i are derived as:

$$p_i = \frac{1/(\rho_{i,\text{Bench}} \cdot \sigma_i)}{\sum_{k=1,k\neq NESG}^{K} 1/(\rho_{k,\text{Bench}} \cdot \sigma_k)}$$

This formula ensures that assets with lower risk (lower σ_i or lower $\rho_{i,\text{B}}$) receive a proportionally larger share of the reallocated weight, thereby equalizing their incremental risk contributions.

Substituting the expression for p_i into the incremental risk contribution equation yields:

$$p_i \cdot w^{NESG} \cdot \rho_{i,\text{Bench}} \cdot \sigma_i = \frac{\frac{1}{\rho_{i,\text{Bench}} \cdot \sigma_i}}{\sum_{k=1,k\neq NESG}^{K} 1/(\rho_{k,\text{Bench}} \cdot \sigma_k)} \cdot w^{NESG} \cdot \rho_{i,\text{Bench}} \cdot \sigma_i$$

Simplifying, we obtain:

$$\Delta Contrib_i = \frac{w^{NESG}}{\sum_{k=1,k\neq NESG}^{N} 1/(\rho_{k,\text{Bench}} \cdot \sigma_k)} \ \forall i$$

This result demonstrates that the incremental risk contribution $\Delta Contrib_i$ is identical for all eligible assets.

In other words, each asset's risk contribution increases by the same amount, ensuring a balanced risk distribution across the ESG-adjusted index.

The risk-parity-based reallocation framework offers three key advantages:

1. Uniform Risk Contribution: Each remaining asset receives an identical increase in risk contribution, mitigating the risk of unintended concentration.

2. Preservation of Risk Structure: The ESG-adjusted benchmark maintains a balanced risk profile.

3. Isolation of ESG Effects: The performance differential between the ESG-adjusted benchmark and the original benchmark reflects the pure impact of ESG constraints, uncontaminated by changes in risk concentration.

This framework provides a rigorous and transparent methodology for assessing the return impact of ESG strategies in benchmark construction. By ensuring that reallocated weights contribute equally to portfolio risk, it enables asset managers and owners to evaluate the true cost or benefit of ESG integration without introducing unintended risk biases. This approach is particularly valuable in practical portfolio management, where clarity and risk control are paramount.

Illustration: Comparative Analysis: Risk Parity vs. Proportional Reallocation

Let us assume that the firm's ESG strategy prohibits investment in the energy sector. Table 2 below presents, in the first two columns, the percentages and marginal risk contributions of the benchmark, which is not subject

Table 2: Comparison of Risk-Parity Allocation and Proportional

	%	Marginal cont	Risk Parity %	New Parity %	New Prop %	Diff Risk Contrib Parity	Diff Risk Contrib Prop	New Contrib	New Contrib Prop
Consumer Discretionary	13.86	10.98	7.20	14.23	14.60	0.04	0.08	1.56	1.60
Consumer Staples	10.51	6.35	12.45	11.13	11.07	0.04	0.04	0.71	0.70
Energy	5.02	7.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Financials	21.77	8.16	9.69	22.26	22.92	0.04	0.09	1.82	1.87
Health Care	7.41	9.89	7.99	7.81	7.80	0.04	0.04	0.77	0.77
Industrials	15.30	9.29	8.51	15.73	16.11	0.04	0.08	1.46	1.50
Information Technology	7.07	5.71	13.84	7.76	7.44	0.04	0.02	0.44	0.42
Materials	8.29	9.57	8.26	8.71	8.73	0.04	0.04	0.83	0.84
Telecom Services	5.35	4.50	17.56	6.23	5.63	0.04	0.01	0.28	0.25
Utilities	5.41	5.44	14.52	6.14	5.70	0.04	0.02	0.33	0.31
Total	**100.00**		**100.00**	**100.00**	**100.00**	**0.36**	**0.42**	**8.21**	**8.27**

to ESG constraints. The third column provides the risk-parity allocations—excluding the energy sector—required to ensure that each sector contributes equally to index risk.

The risk-parity allocation for the Consumer Discretionary sector, for instance, is calculated using Equation 2:

$$w_{\text{Cons Disc}}^{(\rho=a)} = \frac{1/\rho_{i,M}\sigma_i}{\sum_{i=1}^{K} 1/\rho_{i,M}\sigma_i} =$$

$$\frac{1/10.98}{1/10.98 + 1/6.35 + 1/8.16 + 1/9.89 + \cdots + 1/5.44} =$$

$$7.20\%$$

The risk parity weights are then used to reallocate the 5.02**%** originally assigned to the energy sector across the remaining sectors. Consequently, the new allocation for the Consumer Discretionary sector becomes

$$14.23\% = 13.86\% + 0.072 \times 5.02\%$$

A comparison of the allocations obtained through risk parity and proportional methods reveals distinct implications for sector weights: sectors with high initial index weights experience a more substantial increase under the proportional method. For example, the Financials sector increases by 1.15% (22.92 – 21.77) under proportional reallocation but only by 0.49% (22.26 – 21.77) under risk parity. Conversely, sectors with low initial benchmark weights, such as Utilities, see minimal adjustments under the proportional method (0.29%) but a more significant increase under risk parity (0.73%).

Impact on Risk Contribution

The two methods also differ in their impact on risk contribution. The proportional method amplifies the risk contribution of sectors that are already prominent in the benchmark. For Financials, the risk contribution rises by 0.09 under this approach, compared to just 0.04 under risk parity. For Utilities, the proportional method yields a negligible increase in risk contribution (0.02), whereas risk parity ensures a uniform increase (0.04), regardless of the sector's initial weight.

While the proportional method exacerbates the risk concentration in already dominant sectors, the risk parity approach distributes risk contributions more evenly. This ensures that the risk contribution previously attributed to sectors excluded by the ESG strategy is redistributed equally across all remaining sectors. The approach thereby prevents the reinforcement of dominant positions in the unconstrained index, fostering greater alignment with the principle of diversification and mitigating the risk of overconcentration in specific sectors.

SECTION 3: ESG ATTRIBUTION

The objective of ESG attribution is to measure the impact on returns resulting from the integration of a firm or asset owner's ESG strategy. It is important to note that our focus lies on active ESG strategies, which involve the incorporation of constraints specific to fund manager or asset owner. The attribution process identifies the contribution of these constraints to the return.

We assume that the ESG integration strategy involves two main steps: first, the exclusion of certain sectors or segments, such as unconventional weapons or fossil fuels; second, within the eligible sectors, securities are selected based on an internal analysis.

The contribution of the ESG strategy is determined by the successive difference in returns between the risk-parity-adjusted benchmark and the original benchmark. The process begins by reallocating the weights of the excluded sectors to the remaining sectors using a risk parity method. Subsequently, the weights of the excluded securities within each sector are redistributed using the same risk parity approach.

The allocation effect measures the impact on returns from excluding entire sectors or segments, while the selection effect measures the impact of excluding individual securities within those segments.

Exclusion Effect

To calculate the exclusion effect at the sector or segment level, we assume no restrictions on individual securities within each segment. The new weights, $w_{i,B}^{ESG}$, in the ESG-adjusted index are determined using Equation 3 (Section 2).

Let:

$$R_B = \sum_{i=1}^{N} w_{i,B} R_{i,B}$$

be the benchmark return without the ESG exclusion strategy.

$$R_B^{ESG} = \sum_{i=1}^{K} w_{i,B}^{ESG} R_{i,B}$$

be the benchmark return adjusted for the ESG exclusion of sectors or segments, K is the number of authorized sectors under the ESG policy

The exclusion effect is given by:

$$R_B^{ESG} - R_B = \sum_{i=1}^{K} (w_{i,B}^{ESG} - w_{i,B}) R_{i,B} \qquad (4)$$

Illustration

Consider an index with the sector composition shown in Table 3 below. The asset owner instructs the manager not to invest in the Energy sector. The original Energy sector weight is reallocated to the remaining sectors, ensuring an equal incremental risk contribution across all allowed sectors.

As outlined in Section 2, the risk-parity weight for the Consumer Discretionary sector is calculated as:

$$w_{Cons\ Disc}^{Parity} = \frac{1/\rho_{i,M}\sigma_i}{\sum_{i=1}^{K} 1/\rho_{i,M}\sigma_i} =$$

$$\frac{1/10.98}{(1/10.98+1/6.35+1/8.16+1/9.89+\cdots+1/5.44)} = 7.20\%$$

The adjusted weight in the ESG-integrated index is:

$$14.23\% = 13.86\% + 0.072 \times 5.02\%$$

The exclusion effect for the Consumer Discretionary sector is:

$$0.02\% = (0.1423 - 0.1386) \times 5.39\%$$

For the excluded Energy sector, the effect is

$$-0.23\% = (0 - 0.0502) \times 4.66\%$$

The total exclusion effect across all sectors results in –0.08% impact on the index return.

It is important to note that the proportional method would have increased the concentration in the Financials and Industrials sectors, thereby reducing the overall diversification of the portfolio.

Selection Effect

The integration of an ESG strategy involves making deliberate choices at the sector or segment levels. Additionally, within each segment, individual securities are selected based on the ESG criteria defined by the asset owner or firm. This process inherently leads to the exclusion of certain securities that do not meet the established ESG standards.

The percentages of excluded securities are reallocated to the remaining authorized securities within the same sector, while maintaining the sector's overall weight in the index after exclusion. This approach ensures that the sector's representation remains consistent with the post-exclusion allocation.

The redistribution of excluded securities among the remaining securities in the sector is conducted while preserving risk parity. The weight of each authorized

Table 3: Exclusion Effect

	%	B Ret	B correl	B Vol	Marginal cont	B Cont Vol	Parity %	ESG %	ESG return	ESG Effect
Consumer Discretionary	13.86	5.39	0.85	12.94	10.98	1.52	7.20	14.23	5.39	0.02
Consumer Staples	10.51	2.73	0.59	10.74	6.35	0.67	12.45	11.13	2.73	0.02
Energy	5.02	4.66	0.59	12.36	7.24	0.36	0.00	0.00		-0.23
Financials	21.77	2.87	0.69	11.89	8.16	1.78	9.69	22.26	2.87	0.01
Health Care	7.41	3.65	0.74	13.36	9.89	0.73	7.99	7.812	3.65	0.01
Industrials	15.30	4.47	0.93	10.02	9.29	1.42	8.51	15.73	4.47	0.02
Information Technology	7.07	3.58	0.62	9.23	5.71	0.40	13.84	7.76	3.58	0.02
Materials	8.29	4.52	0.90	10.65	9.57	0.79	8.26	8.71	4.52	0.02
Telecom Services	5.35	1.19	0.43	10.40	4.50	0.24	17.56	6.23	1.19	0.01
Utilities	5.41	2.19	0.42	12.83	5.44	0.29	14.52	6.14	2.19	0.02
Total	**100.00**	**3.66**	**1.00**	**8.24**		**8.22**	**100.00**	**100.00**	**3.58**	**-0.08**

security is calculated using the following formula:

$$w_{Asset}^{Parity} = \frac{1/(\rho_{i,M} \cdot \sigma_i)}{\sum_{k \in \text{sector}} (1/(\rho_{k,M} \cdot \sigma_k))}$$

where k represents the authorized securities within the sector.

Example: Reallocation in the Health Care Sector

Consider security 2 (A2) in the Health Care sector, which represents 7.812% of the index after the exclusion of the Energy sector. To determine the new weight of Asset 2, we first calculate its risk-parity-adjusted weight within the sector:

$$w_{Asset\ 2}^{Parity} =$$

$$\frac{1/(0.31 \times 18.86)}{1/(0.31 \times 18.86) + 1/(0.49 \times 17.80) + \cdots + 1/(0.22 \times 19.24)} =$$

$$13.86\%$$

The initial weight of Asset 2 in the index (excluding the Energy sector) is adjusted as follows to reflect the exclusion of the sector Energy:

$$w_{Asset\ 2} =$$

$$0.63\% + 13.86\% \times (7.812\% - 7.41\%) = 0.68\%$$

To determine the final weight of Asset 2 in the ESG index, incorporating both the exclusion and selection strategies, we apply the risk-parity percentages again:

$$w_{Asset\ 2}^{ESG} = 0.68\% + 0.1386 \times 2.26\% = 0.99\%$$

With the new allocation of securities within the Health Care sector, we can compute the sector's return in the ESG-adjusted index:

$$w_{Health\ Care}^{ESG} =$$

$$\frac{(0.99 \times 2.60) + (0.39 \times 1.40) + (0.76 \times 1.05) + \cdots + (0.71 \times 1.35)}{0.68 + 0.39 + 0.76 + \cdots + 0.71} =$$

$$2.87\%$$

The selection effect associated with the ESG-driven security selection for the Health Care sector is calculated as:

Table 4: Selection Effect

		%	B Ret	B correl	B Vol	Marginal cont	Parity %	ESG% ex energy	ESG% ex asset	ESG Ret ex	Asset Effect
Consumer Discret		13.86	5.39	0.85	12.94	10.98	7.20	14.23	14.23	5.39	0.00
Consumer Staples		10.51	2.73	0.59	10.74	6.35	12.45	11.13	11.13	2.73	0.00
Energy		5.02	4.66	0.59	12.36	7.24	0.00	0.00	0.00		0.00
Financials		21.77	2.87	0.69	11.89	8.16	9.69	22.26	22.26	2.87	0.00
Health Care		7.41	3.65	0.74	13.36	9.89	7.99	7.812	7.812	2.87	-0.06
	A 1	*2.26*	*4.70*	*0.50*	*21.20*	*10.56*	*0.00*	*2.26*	*0.00*		
	A 2	*0.63*	*2.60*	*0.31*	*18.66*	*5.72*	*13.86*	*0.68*	*0.99*		
	A 3	*0.15*	*-1.40*	*0.49*	*17.80*	*8.76*	*9.05*	*0.18*	*0.39*		
	A 4	*0.50*	*1.05*	*0.32*	*24.89*	*8.02*	*9.88*	*0.54*	*0.76*		
	A 5	*0.19*	*-0.75*	*0.50*	*18.79*	*9.35*	*8.48*	*0.23*	*0.42*		
	A 6	*0.78*	*3.01*	*0.65*	*18.30*	*11.96*	*6.63*	*0.81*	*0.95*		
	A 7	*0.30*	*5.12*	*0.36*	*18.17*	*6.58*	*12.05*	*0.35*	*0.62*		
	A 8	*0.15*	*4.87*	*0.40*	*21.01*	*8.46*	*9.37*	*0.19*	*0.40*		
	A 9	*0.10*	*2.57*	*0.60*	*26.30*	*15.80*	*5.02*	*0.12*	*0.23*		
	A 10	*2.14*	*4.42*	*0.49*	*22.45*	*10.93*	*7.25*	*2.17*	*2.33*		
	A 11	*0.22*	*1.35*	*0.22*	*19.24*	*4.30*	*18.42*	*0.29*	*0.71*		
Industrials		15.30	4.47	0.93	10.02	9.29	8.51	15.73	15.73	4.47	0.00
Information Tech		7.07	3.58	0.62	9.23	5.71	13.84	7.76	7.76	3.58	0.00
Materials		8.29	4.52	0.90	10.65	9.57	8.26	8.71	8.71	4.52	0.00
Telecom Services		5.35	1.19	0.43	10.40	4.50	17.56	6.23	6.23	1.19	0.00
Utilities		5.41	2.19	0.42	12.83	5.44	14.52	6.14	6.14	2.19	0.00
Total		**100**	**3.66**	**1.00**	**8.24**		100	**100**	**100**	**3.52**	**-0.06**

$$\text{Selection Effect} = 0.07812 \times (2.87\% - 3.65\%) = -0.06\%$$

This negative selection effect indicates a slight reduction in return due to the ESG-driven adjustments in the sector.

We verify that the return of the index integrating the ESG strategy (exclusion and selection) equals the return of the initial index plus the allocation effect and the selection effect.

$$3.52\% = 3.66\% \underbrace{-0.08\%}_{Exclusion\ effect} \underbrace{-0.06\%}_{Selection\ effect}$$

CONCLUSION

The integration of active ESG strategies fundamentally reshapes the investable universe, altering both benchmark allocation and risk profiles. This paper introduces a risk-parity-based reallocation framework as an alternative to traditional proportional methods, ensuring that excluded assets are redistributed in a way that maintains uniform incremental risk contributions across the portfolio.

Risk-parity reallocation prevents excessive concentration in dominant sectors - such as Financials - while preserving diversification. By isolating the exclusion and selection effects, this methodology provides a transparent assessment of ESG's impact on performance, free from unintended risk biases.

This approach equips asset managers with a rigorous tool to balance financial objectives and sustainability goals, ensuring that ESG integration enhances, rather than distorts, portfolio risk-return dynamics.

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ENDNOTES

[1] It is important to note that most asset management firms calculate performance on a daily basis. This means that the percentages resulting from the reallocation of excluded positions are also recalculated daily. This approach allows for the consideration of evolving correlations and enables real-time adjustments, particularly during periods of crisis.

[2] This assumption enables an analytical solution, which simplifies the understanding and readability of the article. However, accounting for dynamic correlations—those that change with the exclusion of certain assets—requires the use of an optimization algorithm to determine the optimal solution.