Integrating Risk and Return: A Unified Approach to Performance Attribution

In a demanding asset management environment where transparency and risk-adjusted efficiency are essential, we introduce a unified framework for performance attribution that integrates both risk and return. This model extends the traditional Brinson approach by incorporating the market price of risk through the Sharpe ratio. It decomposes performance into allocation and selection effects, assessing their efficiency via a risk-adjusted alpha analogous to Jensen's alpha. The framework evaluates whether active decisions truly added value relative to the risks taken. By comparing to Menchero's risk-adjusted performance models, the study highlights their complementarity in measuring risk-adjusted performance. This integrated perspective provides asset managers and investors with a deeper, more actionable understanding of active management.

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INTRODUCTION

In an increasingly demanding asset management environment—where transparency, risk-adjusted performance, and the manager's added value are key requirements—performance attribution analysis has emerged as a crucial tool for oversight and communication. Performance attribution serves a dual purpose. First, it provides a communication tool that allows managers to rigorously explain performance, thereby strengthening trust relationships with clients. Second, it functions as an internal control tool that enables portfolio teams to diagnose components of outperformance (or underperformance) and optimize future decision-making processes (Le Sourd, 2007).

Traditional models, such as those of Brinson, Hood, and Beebower (1986) or Brinson and Fachler (1985), decompose portfolio returns into allocation and selection effects (and/or interaction effect), providing useful insights into the sources of active performance. However, these models assess outcomes purely in return space and do not account for the risk undertaken to achieve them.

This absence of a risk dimension limits their diagnostic power. Active management decisions—whether in allocation or selection—may generate positive excess returns but still be inefficient if they require more risk-taking. From a modern portfolio management perspective, where efficiency is evaluated through the risk–return trade-off, a complete attribution framework must incorporate both components. José Menchero (2006/2007) advanced the use of the information ratio as a methodological framework for explicitly incorporating risk into performance attribution analysis.

Our contribution is to integrate risk into the performance attribution process and to extend it to derive a risk-adjusted performance coefficient—analogous to Jensen's alpha—for both allocation and selection decisions. By linking risk attribution to expected returns via the market price of risk (Sharpe ratio), we establish a framework capable of answering a fundamental question: Did the active decisions deliver sufficient return relative to the additional risk?

This unified risk–return attribution framework enables:

1. The decomposition of portfolio risk into allocation and selection contributions.

2. The assignment of an expected return to each risk contribution using the Sharpe ratio.

3. The calculation of a segment-level and decision-type alpha, directly comparable to the familiar Jensen's alpha.

In doing so, we offer both portfolio managers and institutional stakeholders a richer, more precise diagnostic of active management efficiency.

In the first section, we present the risk attribution model to identify the contribution of allocation and selection decisions to the evolution of the portfolio's risk profile. In the second section, we propose a performance attribution model that integrates both risk and returns to analyzing the efficiency of active management decisions. Finally, in the third section we explain how to implement the risk-return attribution model, and we provide an example. In the final section, we compare our model with Menchero's risk-adjusted attribution framework to underline their mutual consistency.

RISK ATTRIBUTION: METHODOLOGY, AND EMPIRICAL ILLUSTRATION

Risk attribution helps assess the relevance and efficiency of active management decisions. While performance attribution decomposes return sources, risk attribution analyzes how active management decisions—particularly allocation and security selection—impact the overall risk profile of the portfolio. This approach helps determine whether the change in risk induced by active choices is justified by an increase in performance, from a risk-return efficiency perspective.

Allocation and Selection Effects

The total risk of a portfolio (or a benchmark) can be broken down into a weighted sum of risk contributions from each segment (*e.g.*, sectors or asset classes). A segment's contribution to the portfolio's risk depends on three factors: the weight allocated to that segment w_k, its volatility $\sigma_k^{P,B}$, and the correlation between the segment's returns and the returns of the portfolio (or benchmark) $\rho_{P,B,k}$.

Formally, the risk contribution of segment k is equal to

$$w_k \times \rho_{P,B,k} \times \sigma_k^{P,B}$$

where k denotes the segment, and P or B refers to the portfolio or the benchmark, respectively.

To evaluate the impact of active allocation decisions on risk, the portfolio's risk is compared to that of a synthetic portfolio where segment weights are identical to those of the portfolio, but security selection is passive (*i.e.*, the segment risk is equal to the benchmark's).

The difference in risk between this synthetic portfolio and the benchmark provides an estimate of the allocation effect on total risk. The difference between the synthetic and actual portfolio—where both active allocation and selection decisions are implemented—gives the selection effect on risk. We will use the PB index to denote the synthetic portfolio.

Methodologically, the impact of allocation and selection decisions is measured by the differences in risk contributions between these portfolios.

Allocation decision

To isolate allocation decisions, we compare a portfolio with active allocation and passive selection to the benchmark. This portfolio features, on the one hand, segment risk equal to the benchmark and, on the other hand, different correlations between segments and the overall portfolio. The risk difference between these two portfolios reflects the allocation effect.

$$\sigma_{PB} - \sigma_B = \sum_{k=1}^{n} \left(\underbrace{w_k^P \times \rho_{PB,k} - w_k^B \times \rho_{B,k}}_{diversification \quad gain \ or \ loss} \right) \times \sigma_k^B \tag{1}$$

Where w_k^P and w_k^B represent the percentages invested in the portfolio and the benchmark, respectively. $\rho_{B,k}$ and $\rho_{PB,k}$ represent the correlation of segment k with the benchmark and the synthetic portfolio with active allocation. σ_k^B is the volatility of segment k in the benchmark.

Notably, unlike the Brinson return attribution model, over- or under-exposure to a segment is not captured solely by the weight but by the weight multiplied by the correlation. Qian (2006) provides an intuitive interpretation of the weighted sum of correlations, viewing it as a diversification indicator.

In the absence of diversification, the weighted sum of correlations equals 1, whereas full diversification (*i.e.*, specific risks are offset) brings this sum to 0. This interpretation positions the allocation term as a measure of how changes in diversification, due to active allocation decisions, contribute to risk.

Selection Decisions

Selection decisions are analyzed by comparing the actual portfolio's risk with that of the synthetic portfolio reflecting only active allocation. The resulting risk difference captures the effect of selection decisions.

$$\sigma_P - \sigma_{PB} = \sum_{k=1}^{N} w_k^P \times \left(\rho_{P,k}\sigma_k^P - \rho_{PB,k}\sigma_k^B\right) \quad (2)$$

This effect reflects the marginal change in risk contributions between dynamically managed sectors and the synthetic portfolio.

Illustration

The table below presents a sector-based example of risk attribution,[1] derived from an equity portfolio and its benchmark. For each sector, the following information is reported: investment weights (for both the portfolio and the benchmark), returns, volatility, as well as the risk allocation and selection effects.

Consider the example of "Health Care." The portfolio manager implemented a 1.15% underweight (7.39% versus 8.54% in the benchmark), reflecting an active allocation decision. The sector return achieved (-6.29%) was slightly below that of the benchmark (-4.89%). The volatility associated with this segment increased from 10.45% in the benchmark to 16.8% in the portfolio, indicating a higher level of risk arising from a different security selection. The risk attribution analysis shows that the contribution of the allocation decision reduced the portfolio's overall risk by 4 basis points, while the selection decision increased it by 51 basis points. Applying the same methodology across all sectors reveals that allocation decisions contributed 105 basis points, whereas selection decisions contributed 654 basis points. The sum of these effects gives the volatility differential (16.62% - 9.04% = 7.58%).

RISK-RETURN ANALYSIS OF ACTIVE MANAGEMENT DECISIONS

The central question is whether the observed changes in portfolio volatility have been efficient, that is, whether the resulting risk-return relationship is optimal. To ad-

Table 1: Risk Attribution

Sector	% Ptf	% Bench	Ret Ptf	Ret Bench	Risk Ptf	Risk Bench	Risk Alloc	Risk Selec
Consumer Discretionary	10.93	13.39	10.34	3.73	18.42	12.66	-0.34	0.54
Consumer Staples	6.69	10.91	5.14	0.74	10.59	5.03	-0.16	0.22
Energy	0.70	5.65	7.84	10.89	33.98	19.76	-0.48	0.05
Financials	24.77	18.26	12.81	15.77	17.33	14.58	0.58	0.60
Health Care	**7.39**	**8.54**	**-6.29**	**-4.89**	**16.80**	**10.45**	**-0.04**	**0.51**
Industrials	13.14	14.37	-1.77	7.67	21.84	12.11	0.03	1.24
Information Technology	15.16	7.69	12.77	14.01	36.33	21.49	1.35	1.85
Materials	10.65	7.21	6.77	3.72	19.02	10.91	0.40	0.83
Real Estate	0.48	2.20	-9.37	-3.21	34.67	9.84	-0.12	0.09
Telecom Services	4.22	5.11	-12.11	-1.66	16.58	7.06	-0.04	0.27
Utilities	5.88	6.68	7.47	5.94	21.69	13.02	-0.13	0.36
Total	**100.0**	**100.0**	**6.08**	**6.41**	**16.62**	**9.04**	**1.05**	**6.54**

dress this issue, we draw upon modern portfolio theory, as introduced by Markowitz (1952), which establishes a systematic relationship between a portfolio's expected return and its risk. According to this framework, a portfolio is deemed efficient if, for a given level of risk, it maximizes the expected return.

Similarly, the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966), asserts that the expected return of a security, segment, or portfolio is equal to the risk-free rate plus a risk premium. This premium is not based on total risk, but rather on the asset's marginal contribution to the overall risk of the market portfolio. In this framework, the risk premium associated with a given asset or segment is equal to the Sharpe ratio (*i.e.*, the price of risk) multiplied by its marginal risk contribution. An active management decision is therefore considered justified if it generates a positive, risk-adjusted excess return—or alpha.

Risk attribution allows for the identification of the risk contributions associated with active allocation and selection decisions. The CAPM framework, in turn, enables the assignment of an expected return to each of these contributions. By integrating these two approaches, it becomes possible to compute the alpha generated by both allocation and selection decisions.

Risk–Return Attribution Model

The objective is to assess whether the realized return is commensurate with the change in risk exposure. The expected return associated with active management decisions can be inferred from their contribution to portfolio volatility using the Sharpe ratio or the market price of risk. For example, with a Sharpe ratio of 0.4, a 1% increase in volatility contribution (which includes the correlation) should be offset by an increase in return of at least 0.4% (0.4 × 1%). The return of 0.4% can then be compared with the results from Brinson-style performance attribution to determine whether the risk-adjusted performance is positive (outperformance) or negative (underperformance). Such an analysis provides a more nuanced understanding of which active management decisions have contributed efficiently to overall performance.

For instance, an asset management firm may decide to emphasize its asset allocation process if the team responsible for allocation consistently delivers positive alpha. Conversely, if security selection regularly generates positive returns that nonetheless fail to compensate for the additional risk incurred, it may be worth investigating the underlying causes of this inefficiency.

In the Brinson Fachler model, the allocation and selection effects are defined by the following relationships:

$$Allocation\ effect = \sum_{k=1}^{N} \left(w_k^P - w_k^B\right)\left(R_k^B - R^B\right)$$

$$Selection\ effect = \sum_{k=1}^{N} w_k^P \left(R_k^P - R_k^B\right)$$

Where:

- w_k^P and w_k^B represent the percentage weights invested in segment *k* in the portfolio and the benchmark, respectively.
- R_k^P and R_k^B represent the returns of segment *k* in the portfolio and the benchmark, respectively.
- R^B is the total return of the benchmark.

The allocation effect is positive when segment *k* is over- (or under-) weighted and the passive strategy delivers a higher (or lower) return than the benchmark, and vice versa.

The selection effect is positive when the budget allocated to segment *k* (w_k) achieves a return higher than that of the passive strategy.

Measuring the Efficiency of Allocation and Selection Decisions

To evaluate the efficiency of allocation and selection decisions, we express the allocation and selection effects from performance attribution in terms of expected return.

For the selection effect, as it is expressed in term of marginal contribution to risk, we multiply it by the Sharpe ratio (λ), which represents the market price of risk, giving for each segment *k*:

$$Expected\ Selection\ Effect = w_k^P \times \left(\rho_{P,k}\sigma_k^P - \rho_{PB,k}\sigma_k^B\right) \times \lambda \quad (3.a)$$

Combining Equation 3 with the Brinson selection effect, we obtain the coefficient alpha, or risk-adjusted selection,

$$Alpha_{Selection} = Brinson\ Selection\ Effect - Expected\ Selection\ Effect \quad (3.b)$$

For the allocation effect, the expected return from these decisions for each segment k under the BBH model is:

$$Exepcted\ Allocation\ Effect = \left(w_k^P \times \rho_{PB,k} - w_k^B \times \rho_{B,k}\right) \times \sigma_k^B \times \lambda \quad (4.a)$$

Similarly, we calculate the allocation alpha or the risk adjusted allocation effect:

$$Alpha_{Allocation} = Brinson\ Allocation\ Effect - Expected\ Allocation\ Effect \quad (4.b)$$

We can then assess efficiency in terms of both profitability and risk by calculating the difference between the portfolio effects and their expected values. A positive difference means the manager achieved a risk-adjusted return above expectations.

Illustration

In Table 2, the ALLOCATION and SELECTION columns show the Brinson performance attribution results (Column Effect), the risk attribution results (Column Risk Contrib) and the expected return given the contribution to risk assuming a Sharpe Ratio of 0.4 (Column E[Ret]).

The allocation and selection effects were computed daily and subsequently chained using the method proposed by Cariño. Therefore, the allocation and selection effects cannot be inferred solely from the percentages reported in the table below.

The fact that the allocation decision for the discretionary sector contributed to reducing risk by 34 basis points implies that the expected return may be lower by 13 basis points. Given that the Brinson allocation effect is positive and amounts to 7 basis points, this indicates that the allocation decision contributed more than expected. It is therefore a decision that improved the portfolio's

Table 2

This table gives the Brinson allocation and selection effects, the risk attribution effects, and the expected return for a Sharpe Ratio equal to 0.4.

	WEIGHTS		RETURNS		ALLOCATION			SELECTION		
Sector	**% Ptf**	**% Bench**	**Ret Ptf**	**Ret Bench**	**Effect**	**E[Ret]**	**Risk Contrib**	**Effect**	**E[Ret]**	**Risk Contrib**
Consumer Discretionary	10.93	13.39	10.34	3.73	0.07	-0.13	-0.34	0.63	0.21	0.54
Consumer Staples	6.69	10.91	5.14	0.74	0.22	-0.06	-0.16	0.29	0.09	0.22
Energy	0.70	5.65	7.84	10.89	-0.29	-0.19	-0.48	-0.01	0.02	0.05
Financials	24.77	18.26	12.81	15.77	0.51	0.23	0.58	-0.67	0.24	0.60
Health Care	7.39	8.54	-6.29	-4.89	0.14	-0.02	-0.04	-0.15	0.20	0.51
Industrials	13.14	14.37	-1.77	7.67	0.01	0.01	0.03	-1.36	0.50	1.24
Information Technology	15.16	7.69	12.77	14.01	0.41	0.54	1.35	-0.10	0.74	1.85
Materials	10.65	7.21	6.77	3.72	-0.12	0.16	0.40	0.37	0.33	0.83
Real Estate	0.48	2.20	-9.37	-3.21	0.16	-0.05	-0.12	-0.03	0.04	0.09
Telecom Services	4.22	5.11	-12.11	-1.66	0.05	-0.01	-0.04	-0.56	0.11	0.27
Utilities	5.88	6.68	7.47	5.94	-0.01	-0.05	-0.13	0.08	0.14	0.36
Total	**100.0**	**100.0**	**6.08**	**6.41**	**1.16**	**0.42**	**1.05**	**-1.52**	**2.62**	**6.54**

risk–return trade-off. In contrast, an analysis of the Information Technology sector shows that the Brinson allocation effect equals 41 basis points, while —given the 135 basis points rise in risk— it should have reached 54 basis points. Hence, despite being positive, the allocation effect is insufficient to compensate for the additional risk, leading to a deterioration of the portfolio's risk–return profile.

At the overall portfolio level, allocation decisions had an expected return of 42 basis points while the active allocation policy resulted in a contribution of 116 basis points. This demonstrates that the portfolio's risk–return trade-off has improved.

Regarding the stock selection policy, we observe that in many sectors the portfolio delivered returns lower than the expected returns. Overall, selection contributed –152 basis points to active return for an increase in volatility of 6.54 percent. This volatility increase corresponds to an expected return of 262 basis points, which is significantly higher than the realized return.

This analysis shows that, for this portfolio, allocation decisions had a *positive* impact on the return–risk trade-off, whereas selection decisions had a *negative* impact.

The following section outlines how risk–return attribution may be implemented in a multi-period setting characterized by successive allocation and selection decisions. Allocation and selection effects are chained using the method proposed by Cariño, whereas risk attribution is based on historical series of return contributions:[2] instead of return series.

IMPLEMENTATION

To establish a unified model for risk and return attribution, it is essential to account for the fact that the weights allocated to each asset class or portfolio segment vary over time. When these weights change, the risk contribution of segment k of the portfolio is given by the correlation between the segment's contribution to return and the total portfolio return, multiplied by the volatility of that segment's contribution to return:

$$Risk\ Contribution = \rho\left(w_{k,t-1}^{P} \times R_{k,t}^{P}; R_{P,t}\right)\sigma\left(w_{k,t-1}^{P} \times R_{k,t}^{P}\right)$$

The total portfolio risk is thus equal to the sum of the contributions across all segments:

$$\sum_{k=1}^{N} \rho\left(w_{k,t-1}^{P} \times R_{k,t}^{P}; R_{P,t}\right)\sigma\left(w_{k,t-1}^{P} \times R_{k,t}^{P}\right) \quad (5)$$

Using this risk decomposition, we can calculate the allocation and selection effects in risk attribution. The allocation effect assumes that only the investment weights in the segments k vary, while the securities selected within each segment are identical to those in the benchmark. The change in risk associated with allocation decisions is the difference between (i) a synthetic portfolio with the same weights as the portfolio but benchmark returns, and (ii) the benchmark itself.

The allocation effect in risk attribution is thus:

$$\sigma_{PB} - \sigma_{B} = \sum_{k=1}^{n} \Big(\rho\left(w_{k,t-1}^{P} \times R_{k,t}^{B}; R_{PB,t}\right) \times \sigma\left(w_{k,t-1}^{P} \times R_{k,t}^{B}\right) - \rho\left(w_{k,t-1}^{B} \times R_{k,t}^{B}; R_{B,t}\right) \times \sigma\left(w_{k,t-1}^{B} \times R_{k,t}^{B}\right)\Big) \quad (6.a)$$

Here, the subscript PB refers to a synthetic portfolio with portfolio weights and benchmark returns. The allocation effect for each segment k equals the difference between the risk contribution of the synthetic portfolio and that of the benchmark.

The selection effect is defined as the difference between the actual portfolio and the synthetic portfolio. The only difference between these two portfolios lies in the segment returns. The selection effect is given by:

$$\sigma_{P} - \sigma_{PB} = \sum_{k=1}^{N} \Big(\rho\left(w_{k,t-1}^{P} \times R_{k,t}^{P}; R_{P,t}\right) \times \sigma\left(w_{k,t-1}^{P} \times R_{k,t}^{P}\right) - \rho\left(w_{k,t-1}^{P} \times R_{k,t}^{B}; R_{PB,t}\right) \times \sigma\left(w_{k,t-1}^{p} \times R_{k,t}^{B}\right)\Big) \quad (6.b)$$

For each asset class or segment, the selection effect is thus the difference between the risk contribution of the actual portfolio and that of the synthetic portfolio.

Once the risk contributions of active allocation and selection decisions have been isolated, we can associate them with an expected return by multiplying these effects by the market price of risk. The market price of risk, or Sharpe ratio, is therefore a parameter of the model that must be set either by reference to the observed ratio for the review period or by using market standards.

We can thus directly measure the alpha or risk-adjusted return of active allocation and selection decisions:

$$Alpha_{Allocation} = Brinson\ Allocation\ Effect - Expected\ Allocation\ Effect$$

Where

$$Expected\ Allocation\ Effect = \left(\rho\left(w^{P}_{k,t-1} \times R^{B}_{k,t}; R_{B,t}\right) \times \sigma\left(w^{P}_{k,t-1} \times R^{B}_{k,t}\right) - \rho\left(w^{B}_{k,t-1} \times R^{B}_{k,t}; R_{B,t}\right)\sigma\left(w^{B}_{k,t-1} \times R^{B}_{k,t}\right)\right) \times \lambda$$

And

$$Alpha_{Selection} = Brinson\ Selection\ Effect - Expected\ Slection\ Effect$$

Where

$$Expected\ Selection\ effect = \left(\rho\left(w^{P}_{k,t-1} \times R^{P}_{k,t}; R_{P,t}\right) \times \sigma\left(w^{P}_{k,t-1} \times R^{p}_{k,t}\right) - \rho\left(w^{P}_{k,t-1} \times R^{B}_{k,t}; R_{B,t}\right) \times \sigma\left(w^{B}_{k,t-1} \times R^{B}_{k,t}\right)\right) \times \lambda$$

Example

We applied this framework to a portfolio invested in equities using weekly data. Based on a six-month historical series of returns and weights for both the portfolio and the benchmark, we computed allocation and selection effects following the Brinson–Fachler model. We then calculated volatilities and correlations between sectors and the portfolio, the benchmark, and the synthetic portfolio. These data were used to derive risk attribution. Finally, assuming a Sharpe ratio of 0.4, we computed the expected returns of active allocation and selection decisions and compared them to the Brinson effects. The alpha of these decisions is obtained as the difference between these two measures.

The results confirm that risk allocation (1.05%) and selection (6.53%) effects reconcile with the difference in volatility between the portfolio and the benchmark, *i.e.*, 7.58 = 16.62 – 9.04.

Integrating risk into the analysis of performance attribution results changes the overall perspective. For instance, in the *Information Technology* sector, the allocation decision initially appears to be a sound one, contributing 0.41% to active return. However, once we account for the fact that this decision also led to a 1.35% increase in volatility contribution (Table 3), the resulting allocation alpha turns negative. This indicates that the

Table 3

The first three columns are the volatility of the contribution to return for the benchmark B, the portfolio P, and the synthetic portfolio PB. The next three columns give the correlation of the contribution to return B, P, and PB.

	$\sigma\left(w^{P}_{k,t-1} \times R^{B}_{k,t}\right)$			$\rho\left(w^{P}_{k,t-1} \times R^{B}_{k,t}; R_{PB,t}\right)$			***Risk Effect***	
	B	P	PB	B	P	PB	ALLOC	SELEC
Consumer Discret	1.69	1.86	1.29	0.84	0.87	0.84	-0.34	0.54
Consumer Staples	0.55	0.70	0.33	0.68	0.62	0.65	-0.16	0.22
Energy	1.12	0.21	0.12	0.47	0.44	0.37	-0.48	0.05
Financials	2.66	4.13	3.48	0.70	0.74	0.70	0.58	0.60
Health Care	0.89	1.20	0.73	0.35	0.65	0.37	-0.04	0.51
Industrials	1.74	3.26	1.80	0.86	0.85	0.85	0.03	1.24
Information Techno	1.65	5.35	3.18	0.80	0.84	0.84	1.35	1.85
Materials	0.79	2.17	1.24	0.90	0.89	0.89	0.40	0.83
Real Estate	0.22	0.18	0.05	0.72	0.71	0.73	-0.12	0.09
Telecom Services	0.36	0.72	0.31	0.58	0.61	0.55	-0.04	0.27
Utilities	0.87	1.17	0.70	0.73	0.74	0.72	-0.13	0.36
Total	**9.04**	**16.62**	**10.08**				**1.05**	**6.53**

Table 4

This table provides the results for the Brinson return attribution, the expected return for a Sharpe Ratio equal to 0.4, and the alpha of the allocation and selection decisions.

	Brinson		**Expected return**		**ALPHA**	
	Alloc	Selec	Alloc	Selec	Alloc	Selec
Consumer Discretionary	0.07	0.63	-0.13	0.21	0.2	0.42
Consumer Staples	0.22	0.29	-0.06	0.09	0.28	0.2
Energy	-0.29	-0.01	-0.19	0.02	-0.1	-0.03
Financials	0.51	-0.67	0.23	0.24	0.28	-0.91
Health Care	0.14	-0.15	-0.02	0.20	0.16	-0.35
Industrials	0.01	-1.36	0.01	0.50	0	-1.86
Information Technology	0.41	-0.10	0.54	0.74	-0.13	-0.84
Materials	-0.12	0.37	0.16	0.33	-0.28	0.04
Real Estate	0.16	-0.03	-0.05	0.04	0.21	-0.07
Telecom Services	0.05	-0.56	-0.01	0.11	0.06	-0.67
Utilities	-0.01	0.08	-0.05	0.14	0.04	-0.06
Total	**1.16**	**-1.52**	**0.42**	**2.62**	**0.74**	**-4.14**

0.41% return contribution is insufficient to compensate for the additional risk taken.

Conversely, in the *Consumer Staples* sector, the allocation contributed 0.22% while simultaneously reducing the risk contribution by 0.16%. This decision therefore enhances the portfolio's efficiency, as reflected by a positive allocation alpha of 0.28 percent.

It is important to note that the alpha reported here has the same interpretation as Jensen's alpha for an asset class. Indeed, Jensen's alpha is computed by comparing realized returns with expected returns, the latter being equal to the market price of risk (Sharpe ratio) multiplied by the asset class's risk contribution.

Comparison with Menchero risk-adjusted attribution[3]

The risk-adjusted attribution model developed by Menchero (2007) allocates the information ratio across the various active management decisions. The risk-adjusted effect depends on the ratio between the attribution effect and the volatility of that effect, weighted by a coefficient that reflects the correlation between the effect and the active return. This coefficient captures the impact of diversification.

When aggregating the different effects, the weighting coefficients are based on the contribution of each active management decision to total risk, rather than on the degree of over- or underweighting.

For comparison purposes, we calculated the allocation and selection components of the information ratio using the same portfolio as in Tables 3 and 4.

Table 5 presents the volatility of the allocation and selection effects, together with the stand-alone Information Ratio.

In Table 6, we report the allocation and selection Information Ratios obtained by multiplying the stand-alone Information Ratio by

$$1/\rho\left(Effect_{k,t}; Active\ Return_{k,t}\right),$$

which can be interpreted as a diversification benefit. We also present the corresponding risk-based weights. The portfolio's overall Information Ratio, calculated over the period, is negative and equal to –0.06. We verify that this portfolio Information Ratio equals the weighted sum of the component Information Ratios, provided that the weights are risk-adjusted, *i.e.,*

$$0.8777 \times 1.59 - 0.1223 \times 0.29 = -0.06.$$

The columns IR show the Absolute Information Ratio adjusted for the diversification benefit (Menchero 2007).

Table 5

	Volatility		Brinson Attribution		Stand Alone IR	
Sector	Alloc	Selec	Alloc	Selec	Alloc	Selec
Consumer Discretionary	0.15	0.77	0.07	0.63	0.48	0.82
Consumer Staples	0.20	0.32	0.22	0.29	1.12	0.90
Energy	0.62	0.10	-0.29	-0.01	-0.47	-0.15
Financials	0.41	1.21	0.51	-0.67	1.24	-0.56
Health Care	0.11	0.73	0.14	-0.15	1.26	-0.20
Industrials	0.03	1.23	0.01	-1.36	0.22	-1.11
Information Technology	0.77	1.72	0.41	-0.10	0.54	-0.06
Materials	0.14	0.79	-0.12	0.37	-0.81	0.47
Real Estate	0.08	0.09	0.16	-0.03	1.92	-0.36
Telecom Services	0.04	0.38	0.05	-0.56	1.28	-1.49
Utilities	0.08	0.47	-0.01	0.08	-0.16	0.16
Total	**1.14**	**5.30**	**1.16**	**-1.52**		

Table 6

	Risk Weights		IR		Contribution to IR	
Sector	Alloc	Selec	Alloc	Selec	Alloc	Selec
Consumer Discretionary	-0.57	8.30	-2.11	1.28	0.01	0.11
Consumer Staples	1.98	2.66	1.90	1.81	0.04	0.05
Energy	2.24	0.50	-2.19	-0.49	-0.05	0.00
Financials	-1.79	15.66	-4.80	-0.72	0.09	-0.11
Health Care	0.27	4.73	8.79	-0.52	0.02	-0.02
Industrials	0.24	12.71	0.49	-1.79	0.00	-0.23
Information Technology	8.34	24.10	0.83	-0.07	0.07	-0.02
Materials	1.02	9.28	-1.93	0.67	-0.02	0.06
Real Estate	0.09	0.97	31.69	-0.58	0.03	-0.01
Telecom Services	0.32	3.51	2.56	-2.67	0.01	-0.09
Utilities	0.09	5.34	-2.26	0.24	0.00	0.01
Total	**12.23**	**87.77**	**1.59**	**-0.29**	**0.19**	**-0.25**

The fact that the correlation between the consumer allocation effect and the active return is negative explains why the ratio is negative, even though the Brinson allocation effect is positive. The last two columns show each sector's contribution to the portfolio's information ratio. We can see that for the Consumer Discretionary sector, the allocation contribution is positive while the information ratio is negative. This can be explained by the fact that the portfolio is short in terms of risk in this sector. Table 7 compares the results of the two models, highlighting their complementarity.

We observe that for almost all sectors, the signs of the Information Ratio and the alpha of both allocation and selection effects are identical. The only exceptions concern the allocation in the Information Technology sector and the selection effects in the Energy and Utilities sectors.

Table 7				
	Contribution to IR		ALPHA	
Sector	Alloc	Selec	Alloc	Selec
Consumer Discretionary	0.01	0.11	0.2	0.42
Consumer Staples	0.04	0.05	0.28	0.2
Energy	-0.05	0	-0.1	-0.03
Financials	0.09	-0.11	0.28	-0.91
Health Care	0.02	-0.02	0.16	-0.35
Industrials	0	-0.23	0	-1.86
Information Technology	0.07	-0.02	-0.13	-0.84
Materials	-0.02	0.06	-0.28	0.04
Real Estate	0.03	-0.01	0.21	-0.07
Telecom Services	0.01	-0.09	0.06	-0.67
Utilities	0	0.01	0.04	-0.06
Total	**0.19**	**-0.25**	**0.74**	**-4.14**

Let us take the allocation effect as an example to explain this difference in sign. The contribution to the Information Ratio depends on the Brinson effect, the correlation, and the risk-adjusted weight. If these components are positive, the sign of the allocation contribution to the Information Ratio will also be positive.

On the other hand, alpha will be positive only if the effect is strong enough to compensate for the additional risk taken. Alpha represents a return, whereas the Information Ratio is a dimensionless measure. While alpha can be directly interpreted as a risk-adjusted return, the contribution to the Information Ratio requires a reference (which may be zero) to determine whether the contribution is positive or not.

To draw a parallel, we could refer to the Modigliani–Modigliani (M^2) coefficient, which transforms the Sharpe ratio into a benchmark risk-adjusted return.

CONCLUSION

This article has proposed a unified framework for performance attribution that explicitly integrates risk into the analysis of active management decisions. By decomposing portfolio risk into allocation and selection components and linking these to expected returns via the market price of risk, we extend traditional attribution models to produce a decision-specific alpha—analogous to Jensen's alpha—at both the allocation and selection levels.

The introduction of this alpha coefficient transforms the interpretation of attribution results. While conventional return-only models may show positive contributions from allocation or selection, the risk-adjusted model can reveal whether these contributions were achieved efficiently. In our empirical illustration, selection decisions that appeared favourable in the Brinson framework were shown, once adjusted for risk, to be inefficient.

The practical implication is clear: by embedding the risk dimension into attribution, asset managers gain a more complete and actionable measure of active decision quality. This allows them to validate successful processes, identify inefficient risk-taking, and refine their investment approach. For stakeholders and clients, the integration of risk-adjusted alpha enhances transparency and credibility, aligning performance evaluation with the core principles of modern portfolio theory.

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ENDNOTES

[1] All the examples in this article are based on a portfolio over the period from June 30, 2022, to December 31, 2022. We would like to thank Amindis for providing the data and software used to perform all the calculations. The raw data are available upon request from the author.

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